Problems and Solutions Section 1.4 (1.40 through 1.56)

1.40  A spring-mass-damper system has mass of 100 kg, stiffness of 3000 N/m and damping coefficient of 300 kg/s. Calculate the undamped natural frequency, the damping ratio and the damped natural frequency. Does the solution oscillate?

Solution: Working straight from the definitions:

\[ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3000 \text{ N/m}}{100 \text{ kg}}} = 5.477 \text{ rad/s} \]

\[ \zeta = \frac{c}{2 \sqrt{km}} = \frac{300}{2 \sqrt{(3000)(100)}} = 0.274 \]

Since \( \zeta \) is less than 1, the solution is underdamped and will oscillate. The damped natural frequency is \( \omega_d = \omega_n \sqrt{1 - \zeta^2} = 5.27 \text{ rad/s} \).

1.41  A spring-mass-damper system has mass of 150 kg, stiffness of 1500 N/m and damping coefficient of 200 kg/s. Calculate the undamped natural frequency, the damping ratio and the damped natural frequency. Is the system overdamped, underdamped or critically damped? Does the solution oscillate?

Solution: Working straight from the definitions:

\[ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1500 \text{ N/m}}{150 \text{ kg}}} = 3.162 \text{ rad/s} \]

\[ \zeta = \frac{c}{2 \sqrt{km}} = \frac{200}{2 \sqrt{(1500)(150)}} = 0.211 \]

Since \( \zeta \) is less than 1, the solution is underdamped and will oscillate. The damped natural frequency is \( \omega_d = \omega_n \sqrt{1 - \zeta^2} = 3.091 \text{ rad/s} \).